Plane

1 Mark Questions

1. Write the vector equation of the plane passing through the point (a, b, c) and parallel to the plane $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 2$. **Delhi 2014**

The required plane is passing through the point (a,b,c) whose position vector is

$\vec{p} = a\hat{i} + b\hat{j} + c\hat{k}$ and is parallel to the plane

$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 2$.

So, it is normal to the vector

$$\vec{n} = \hat{i} + \hat{j} + \hat{k}$$

Hence, required equation of plane is

$$(\vec{r} - \vec{p}) \cdot \vec{n} = 0$$

$$\Rightarrow \quad \vec{r} \cdot \vec{n} = \vec{p} \cdot \vec{n}$$

$$\Rightarrow \quad \vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = (a\hat{i} + b\hat{j} + c\,\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k})$$

$$\Rightarrow \quad \vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = a + b + c$$ **(1)**

2. Find the length of the perpendicular drawn from the origin to the plane $2x-3y+6z+21=0$.

All India 2013

> The distance from point (x_1, y_1, z_1) to the plane $Ax+By+Cz+D=0$ is $\dfrac{Ax_1+By_1+Cz_1+D}{\sqrt{A^2+B^2+C^2}}$.

Given, equation of plane is

$$2x-3y+6z+21=0 \quad \text{...(i)}$$

$\therefore$ Length of the perpendicular drawn from the origin to this plane

$$=\left|\frac{2\cdot 0-3\cdot 0+6\cdot 0+21}{\sqrt{(2)^2+(-3)^2+(6)^2}}\right|=\left|\frac{0-0+0+21}{\sqrt{4+9+36}}\right|$$

$$=\frac{21}{\sqrt{49}}=\frac{21}{7}$$

$$=3 \text{ units}$$ **(1)**

3. Find distance of the plane $3x-4y+12z=3$ from the origin.

Delhi 2011

Given equation of plane is $3x-4y+12z-3=0$ and the point is $(0,0,0)$. We know that, distance of the plane $Ax+By+Cz+D=0$ from the point (x_1, y_1, z_1) is

$$d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

Here, $x_1 = y_1 = z_1 = 0$

and $A = 3, B = -4, C = 12, D = -3$

$\therefore$ Required distance

$$d = \frac{|3(0) - 4(0) + 12(0) - 3|}{\sqrt{(3)^2 + (-4)^2 + (12)^2}}$$

$$\Rightarrow \quad d = \frac{|-3|}{\sqrt{9 + 16 + 144}} = \frac{3}{\sqrt{169}} = \frac{3}{13} \text{ units}$$ **(1)**

4. Write the intercept cut-off by plane $2x + y - z = 5$ on X-axis. **HOTS; Delhi 2011**

> Firstly, we convert the given equation of plane in intercept form, i.e. $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, which cut the X-axis at $(a, 0, 0)$.

Given equation of plane is $2x + y - z = 5$.

On dividing both sides by 5, we get

$$\frac{2x}{5} + \frac{y}{5} - \frac{z}{5} = 1$$

$$\Rightarrow \quad \frac{x}{\left(\frac{5}{2}\right)} + \frac{y}{5} + \frac{z}{(-5)} = 1$$

On comparing above equation of plane with the intercept form of equation of plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

where, a = x-intercept, b = y-intercept and c = z-intercept

We get, $\quad a = \frac{5}{2}$

i.e. intercept cut-off on X-axis $= \frac{5}{2}$ **(1)**

5. Write the distance of following plane from origin, $2x - y + 2z + 1 = 0$. **All India 2010**

Do same as Que. 3 $\left[\textbf{Ans. } \frac{1}{3} \text{ unit}\right]$

6. Find the value of λ, such that the line $\frac{x-2}{6} = \frac{y-1}{\lambda} = \frac{z+5}{-4}$ is perpendicular to the plane $3x - y - 2z = 7$. **All India 2010C**

Given, line $\frac{x-2}{6} = \frac{y-1}{\lambda} = \frac{z+5}{-4}$ is perpendicular to plane $3x - y - 2z = 7$.

Therefore, DR's of the line are proportional to the DR's normal to the plane.

$$\therefore \quad \frac{6}{3} = \frac{\lambda}{-1} = \frac{-4}{-2} \Rightarrow 2 = -\lambda$$

$$\Rightarrow \quad \lambda = -2 \quad \textbf{(1)}$$

4 Marks Questions

7. A plane makes intercepts $-6, 3, 4$ respectively on the coordinate axes. Find the length of the perpendicular from the origin on it. **Delhi 2014C**

Given, intercepts on the coordinate axes are (–6, 3, 4), then equation of plane will be

$$\frac{x}{-6}+\frac{y}{3}+\frac{z}{4}=1$$

or $$\frac{x}{-6}+\frac{y}{3}+\frac{z}{4}-1=0 \quad \textbf{(1)}$$

Distance of a point from plane is given by

$$D=\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right| \quad \textbf{(1)}$$

Here, distance of origin from above plane

$$=\left|\frac{\left(\frac{-1}{6}\right)\cdot 0+\left(\frac{1}{3}\right)\cdot 0+\left(\frac{1}{4}\right)\cdot 0-1}{\sqrt{\left(\frac{-1}{6}\right)^2+\left(\frac{1}{3}\right)^2+\left(\frac{1}{4}\right)^2}}\right|$$

$$=\left|\frac{-1}{\sqrt{\frac{1}{36}+\frac{1}{9}+\frac{1}{16}}}\right|=\left|\frac{-1}{\sqrt{\frac{29}{144}}}\right|=\frac{12}{\sqrt{29}} \quad \textbf{(1)}$$

Hence, required length of the perpendicular from origin to plane is $\frac{12}{\sqrt{29}}$ units. **(1)**

8. Find the distance of the point (–1, – 5, – 10) from the point of intersection of the line $\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda(3\hat{i}+4\hat{j}+2\hat{k})$ and the plane $\vec{r}\cdot(\hat{i}-\hat{j}+\hat{k})=5$.

Delhi 2014; All India 2014C, 2011; HOTS

Firstly, convert the given equations of line and plane in cartesian form and then solve them to get their point of intersection, then use distance formula to find the required distance.

Given equations of line and plane are

$$\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$$

and $$\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$$

Above equations in cartesian form can be written as

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2} \quad \text{...(i)}$$

and $$x - y + z = 5 \quad \text{...(ii)}$$ **(1)**

Let the point of intersection of line (i) and plane (ii) be Q.

Let $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}=\lambda$

$\Rightarrow \quad \frac{x-2}{3}=\lambda, \frac{y+1}{4}=\lambda, \frac{z-2}{2}=\lambda$

$\Rightarrow \quad x=3\lambda+2, y=4\lambda-1, z=2\lambda+2$

$\therefore$ Any point Q on the given line is

$Q(3\lambda+2, 4\lambda-1, 2\lambda+2)$ **(1)**

Since, plane also passes through Q, so coordinates of Q will satisfy Eq. (ii).

$\therefore \quad (3\lambda+2)-(4\lambda-1)+(2\lambda+2)=5$

$\Rightarrow \quad 3\lambda+2-4\lambda+1+2\lambda+2=5$

$\Rightarrow \quad \lambda=0$ **(1)**

On putting $\lambda=0$ in $Q(3\lambda+2, 4\lambda-1, 2\lambda+2)$, we get the point of intersection as $Q(2,-1,2)$.

Now, the required distance

$PQ=\sqrt{(2+1)^2+(-1+5)^2+(2+10)^2}=13$

$\left[\because \text{distance}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}\right]$

$=\sqrt{9+16+144}$

$=\sqrt{144+25}$

$=\sqrt{169}$

$=13$ units

Hence, the required distance is 13 units. **(1)**

9. Show that the lines $\frac{5-x}{-4}=\frac{y-7}{4}=\frac{z+3}{-5}$ and $\frac{x-8}{7}=\frac{2y-8}{2}=\frac{z-5}{3}$ are coplanar. **Delhi 2014**

Given lines can be written as

$$\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$$

and $$\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$$ **(1)**

On comparing both lines with,

$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{b}$ respectively, we get

$x_1=5, y_1=7, z_1=-3, a_1=4, b_1=4, c_1=-5$

and $x_2=8, y_2=4, z_2=5, a_2=7, b_2=1,$

$c_2=3$ **(1)**

If given lines are coplanar, then

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$ **(1)**

$$\text{LHS}=\begin{vmatrix} 8-5 & 4-7 & 5+3 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix}=\begin{vmatrix} 3 & -3 & 8 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix}$$

$$=3(12+5)+3(12+35)+8(4-28)$$

$$=3\times17+3\times47+8(-24)$$

$$=51+141-192=192-192=0=\text{RHS}$$

Therefore, given lines are coplanar. **(1)**

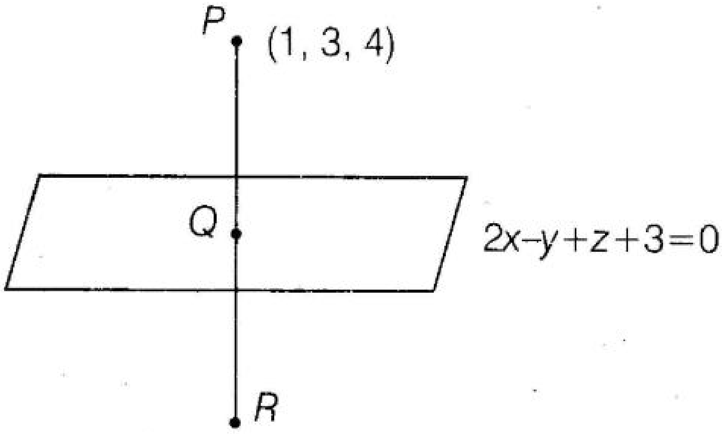
10. Find the image of the point having position vector $\hat{i}+3\hat{j}+4\hat{k}$ in the plane

$\vec{r}(2\hat{i}-\hat{j}+\hat{k})+3=0.$ **All India 2014C**

Given, position vector of point is $(\hat{i}+3\hat{j}+4\hat{k})$.

So, coordinates of point P are (1, 3, 4) and vector equation of plane is $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})+3=0$, then cartesian equation of plane is $2x-y+z+3=0$.

Let Q be the foot of perpendicular from P on the plane.



(1)

Since, PQ is perpendicular to the plane. Hence, Dr's of PQ will be 2, −1, 1.

So, equation of PQ will be

$$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = \lambda \qquad \text{[say]}$$

Coordinates of $Q = (2\lambda + 1, -\lambda + 3, \lambda + 4)$, also Q lies on plane, so it will satisfy the equation of plane.

$$\therefore \quad 2(2\lambda + 1) - 1(-\lambda + 3) + (\lambda + 4) + 3 = 0$$

$$\Rightarrow \quad 4\lambda + 2 + \lambda - 3 + \lambda + 4 + 3 = 0$$

$$\Rightarrow \quad 6\lambda = -6$$

$$\Rightarrow \quad \lambda = -1$$ **(1)**

So, coordinates of Q will be $(-1, 4, 3)$. Since, Q is the mid-point of PR and let R be (x, y, z),

then $\left(\frac{x+1}{2}, \frac{y+3}{2}, \frac{z+4}{2}\right) \equiv (-1, 4, 3)$ **(1)**

On comparing corresponding coordinates, we get

$$\frac{x+1}{2} = -1 \Rightarrow x = -2 \Rightarrow x = -3,$$

$$\frac{y+3}{2} = 4 \Rightarrow y + 3 = 8 \Rightarrow y = 8 - 3 = 5$$

and $\frac{z+4}{2} = 3 \Rightarrow z + 4 = 6 \Rightarrow z = 6 - 4 = 2$

Hence, required coordinates of image point R is $(-3, 5, 2)$. **(1)**

11. Find the vector equation of the plane through the points $(2, 1 - 1)$ and $(-1, 3, 4)$ and perpendicular to the plane $x - 2y + 4z = 10$.

All India 2013

The required plane passes through two points $P(2, 1, -1)$ and $Q(-1, 3, 4)$. Let $\vec{a}$ and $\vec{b}$ be the position vectors of points P and Q, respectively.

Then, $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$

and $\vec{b} = -\hat{i} + 3\hat{j} + 4\hat{k}$

$\therefore \quad \overrightarrow{PQ} = \vec{b} - \vec{a} = (-\hat{i} + 3\hat{j} + 4\hat{k}) - (2\hat{i} + \hat{j} - \hat{k})$

$= -3\hat{i} + 2\hat{j} + 5\hat{k}$ **(1)**

Let $\vec{n_1}$ be the normal vector to the given plane, $x - 2y + 4z = 10$, then $\vec{n_1} = \hat{i} - 2\hat{j} + 4\hat{k}$.

Let $\vec{n}$ be the normal vector to the required plane. Then,

$$\vec{n} = \vec{n_1} \times \overrightarrow{PQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 4 \\ -3 & 2 & 5 \end{vmatrix}$$

$= \hat{i}(-10 - 8) - \hat{j}(5 + 12) + \hat{k}(2 - 6)$

$= -18\hat{i} - 17\hat{j} - 4\hat{k}$ **(1)**

The required plane passes through a point having position vector $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and is normal to the vector $\vec{n_1} = -18\hat{i} - 17\hat{j} - 4\hat{k}$. So, its vector equation is

$\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$ **(1)**

$\Rightarrow \quad \vec{r} \cdot (-18\hat{i} - 17\hat{j} - 4\hat{k}) = (2\hat{i} + \hat{j} - \hat{k}) \cdot (-18\hat{i} - 17\hat{j} - 4\hat{k})$

$\Rightarrow \quad \vec{r} \cdot (-18\hat{i} - 17\hat{j} - 4\hat{k}) = -36 - 17 + 4$

$\therefore \quad \vec{r} \cdot (18\hat{i} + 17\hat{j} + 4\hat{k}) = 49$ **(1)**

12. Find the coordinates of the point, where the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}$ intersects the plane $x-y+z-5=0$. Also, find the angle between the line and the plane. **Delhi 2013**

Given equation of line is

$$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}=\lambda \quad \text{[say]}$$

$$x=3\lambda+2, y=4\lambda-1, z=2\lambda+2$$

Then, $[(3\lambda+2),(4\lambda-1),(2\lambda+2)]$ be any point on the given line. **(1)**

This point lies on the plane $x-y+z-5=0$

$\therefore \quad (3\lambda+2)-(4\lambda-1)+(2\lambda+2)-5=0$

$\Rightarrow \quad 3\lambda+2-4\lambda+1+2\lambda+2-5=0$

$\Rightarrow \quad \lambda=0$ ...(i) **(1)**

$\therefore$ Point of intersection of line and the plane

$=(3\times0+2, 4\times0-1, 2\times0+2)=(2,-1,2)$ **(1/2)**

Let θ be the angle between line and plane.

Then,

$$\sin\theta=\frac{al+bm+cn}{\sqrt{a^2+b^2+c^2}\sqrt{l^2+m^2+n^2}}$$

Here, $a=3, b=4, c=2; l=1, m=-1, n=1.$

$$\therefore \quad \sin\theta=\frac{(3)(1)+4(-1)+2(1)}{\sqrt{9+16+4}\sqrt{1+1+1}}$$

$$\Rightarrow \quad \sin\theta=\frac{3-4+2}{\sqrt{29}\sqrt{3}}=\frac{1}{\sqrt{87}}$$

$$\Rightarrow \quad \theta=\sin^{-1}\frac{1}{\sqrt{87}}$$ **(1½)**

which is the required angle.

13. Find the vector equation of the plane which contains the line of intersection of the planes $\vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k})-4=0$ and $\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})+5=0$ and which is perpendicular to the plane $\vec{r}\cdot(5\hat{i}+3\hat{j}-6\hat{k})+8=0.$ **HOTS; Delhi 2013**

Let the given equation of plane in cartesian form be $\pi_1 \equiv x+2y+3z-4=0$

and $\pi_2 \equiv 2x+y-z+5=0$

Equation of plane through π_1 and π_2 is

$$(x+2y+3z-4)+k(2x+y-z+5)=0$$

$$\Rightarrow x(1+2k)+y(2+k)+z(3-k)-4+5k=0 \quad ...(i)$$

(1)

This plane is perpendicular to the given plane $\vec{r}\cdot(5\hat{i}+3\hat{j}-6\hat{k})+8=0$ whose cartesian equation is

$$5x+3y-6z+8=0$$

$$\therefore \quad 5(1+2k)+3(2+k)-6(3-k)=0$$

$$[\because l_1l_2+m_1m_2+n_1n_2=0]$$

$$\Rightarrow \quad 5+10k+6+3k-18+6k=0$$

$$\Rightarrow \quad 19k-7=0 \Rightarrow k=\frac{7}{19} \quad \textbf{(2)}$$

On putting $k=\frac{7}{19}$ in Eq. (i), we get the equation of plane as

$$x\left(1+\frac{14}{19}\right)+y\left(2+\frac{7}{19}\right)+z\left(3-\frac{7}{19}\right)-4+\frac{35}{19}=0$$

$$\Rightarrow x\left(\frac{33}{19}\right)+y\left(\frac{45}{19}\right)+z\left(\frac{50}{19}\right)-\frac{41}{19}=0$$

$$\Rightarrow \quad 33x+45y+50z-41=0 \quad \textbf{(1)}$$

14. Find the equation of plane(s) passing through the intersection of planes $x + 3y + 6 = 0$ and $3x - y - 4z = 0$ and whose perpendicular distance from origin is unity.

HOTS; All India 2010C

> Firstly, write the required equation of plane as $(x + 3y + 6) + \lambda(3x - y - 4z) = 0$.
> Then, convert the above equation in general form of plane which is $ax + by + c + d = 0$. Finally, use the formula for distance from a point (x_1, y_1, z_1) to the plane $ax + by + cz + d = 0$.

Let the required equation of plane passing through the intersection of planes $x + 3y + 6 = 0$ and $3x - y - 4z = 0$ be

$$(x + 3y + 6) + \lambda (3x - y - 4z) = 0 \quad \ldots(i)$$

Above equation can be written as

$$x + 3y + 6 + 3\lambda x - \lambda y - 4\lambda z = 0$$

$$\Rightarrow x(1 + 3\lambda) + y(3 - \lambda) - 4\lambda z + 6 = 0 \quad \ldots(ii) \; \textbf{(1)}$$

which is the general form of equation of plane.

Also, given that perpendicular distance of plane (i) from origin, i.e. (0, 0, 0) is unity, i.e. one.

$$\therefore \left| \frac{(1 + 3\lambda)(0) + (3 - \lambda)(0) - 4\lambda(0) + 6}{\sqrt{(1 + 3\lambda)^2 + (3 - \lambda)^2 + (-4\lambda)^2}} \right| = 1$$

$\Big[\because$ distance of point (x_1, y_1, z_1) from a plane $ax + by + cz + d = 0$ is given by

$$d = \left| \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}} \right|$$

here, $a = 1 + 3\lambda, b = 3 - \lambda, c = -4\lambda$, $(x_1, y_1, z_1) = (0, 0, 0)\Big]$ **(1)**

$$\Rightarrow \left|\frac{6}{\sqrt{1+9\lambda^2+6\lambda+9+\lambda^2-6\lambda+16\lambda^2}}\right|=1$$

$$\Rightarrow \quad \frac{6}{\sqrt{26\lambda^2+10}}=1$$

$$\Rightarrow \quad 6=\sqrt{26\lambda^2+10}$$

On squaring both sides, we get

$$36 = 26\lambda^2 + 10$$

$$\Rightarrow \quad 26\lambda^2 = 26 \quad \Rightarrow \quad \lambda^2 = 1 \quad \Rightarrow \quad \lambda = \pm 1 \quad \textbf{(1)}$$

Now, on putting $\lambda = 1$ in Eq. (i), we get

$$x + 3y + 6 + 3x - y - 4z = 0$$

$$\Rightarrow \quad 4x + 2y - 4z + 6 = 0$$

$$\Rightarrow \quad 2x + y - 2z + 3 = 0 \quad \text{...(iii)}$$

Again, on putting $\lambda = -1$ in Eq. (i), we get

$$x + 3y + 6 - 3x + y + 4z = 0$$

$$\Rightarrow \quad -2x + 4y + 4z + 6 = 0$$

$$\Rightarrow \quad x - 2y - 2z - 3 = 0 \quad \text{...(iv)}$$

Eqs. (iii) and (iv) are the required equations of the plane. **(1)**

15. Find the equation of plane passing through the point $A(1, 2, 1)$ and perpendicular to the line joining points $P(1, 4, 2)$ and $Q(2, 3, 5)$. Also, find distance of this plane from the line $\frac{x+3}{2}=\frac{y-5}{-1}=\frac{z-7}{-1}$. **Delhi 2010C**

Equation of plane passing through the point $A(1, 2, 1)$ is given as

$$a(x-1) + b(y-2) + c(z-1) = 0 \quad \text{...(i)} \quad \textbf{(1)}$$

[∵ equation of plane passing through (x_1, y_1, z_1) having DR's a, b, c is $a(x-x_1) + b(y-y_1) + c(z-z_1) = 0$]

Now, DR's of line PQ, where $P(1, 4, 2)$ and

Q(2, 3, 5) are 2 − 1, 3 − 4, 5 − 2, i.e. 1, − 1, 3.

Since, plane (i) is perpendicular to line PQ.

∴ DR's of plane (i) are 1, −1, 3

i.e. $a = 1, b = -1, c = 3$ **(1)**

On putting values of a, b and c in Eq. (i), we get the required equation of plane as

$$1(x-1) - 1(y-2) + 3(z-1) = 0$$

$$\Rightarrow \quad x - 1 - y + 2 + 3z - 3 = 0$$

$$\Rightarrow \quad x - y + 3z - 2 = 0 \quad \text{...(ii)}$$

Now, the given equation of line is

$$\frac{x+3}{2} = \frac{y-5}{-1} = \frac{z-7}{-1} \quad \text{...(iii)} \; \mathbf{(1)}$$

DR's of this line are (2, −1, −1) and point (−3, 5, 7).

Now, From Eqs. (ii) and (iii), we get

$$\therefore \quad 2(1) - 1(-1) - 1(3) = 2 + 1 - 3 = 0$$

[by using $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$]

So, line (iii) is parallel to plane (i).

∴ The required distance = Distance of the point (−3, 5, 7) from the plane (ii)

$$\Rightarrow \quad d = \left| \frac{(-3)(1) + (5)(-1) + 7(3) - 2}{\sqrt{(1)^2 + (-1)^2 + (3)^2}} \right|$$

[∵ distance of the point (x_1, y_1, z_1) to the plane $ax + by + cz + d = 0$ is $d = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$]

$$= \left| \frac{-3 - 5 + 21 - 2}{\sqrt{1+1+9}} \right| = \left| \frac{11}{\sqrt{11}} \right| = \left| \frac{(\sqrt{11})^2}{\sqrt{11}} \right|$$

$= \sqrt{11}$ units **(1)**

16. Find the cartesian equation of the plane passing through points $A(0, 0, 0)$ and $B(3, -1, 2)$ and parallel to the line $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$. HOTS; Delhi 2010

> The equation of any plane passing through (x_1, y_1, z_1) is
>
> $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0.$
>
> This plane is parallel to the line
>
> $$\frac{x - x_2}{a_1} = \frac{y - y_2}{b_1} = \frac{z - z_2}{c_1}.$$
>
> $\therefore$ Normal to the plane is perpendicular to the line, i.e. $aa_1 + bb_1 + cc_1 = 0$. Use these results and solve it.

Equation of plane passing through the point $A(0, 0, 0)$ is

$$a(x - 0) + b(y - 0) + c(z - 0) = 0$$

$$\Rightarrow \quad ax + by + cz = 0 \quad \text{...(i)} \textbf{(1)}$$

[using one point form of plane

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$]

Given, the plane (i) passes through the point $B(3, -1, 2)$.

$\therefore$ Put $x = 3$, $y = -1$ and $z = 2$ in Eq. (i), we get

$$3a - b + 2c = 0 \quad \text{...(ii)}$$

Also, the plane (i) is parallel to the line

$$\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$$

$$\therefore \quad a(1) + b(-4) + c(7) = 0$$

[if plane is parallel to the line, then normal to the plane is perpendicular to the line.
$\therefore a_1a_2 + b_1b_2 + c_1c_2 = 0$]

$$\Rightarrow \quad a - 4b + 7c = 0 \quad \text{...(iii)} \textbf{(1)}$$

Now, on multiplying Eq. (iii) by 3 and subtracting it from Eq. (ii), we get

$$\begin{array}{r} 3a - b + 2c = 0 \\ 3a - 12b + 21c = 0 \\ - \quad + \quad\quad - \\ \hline 11b - 19c = 0 \end{array}$$

$\Rightarrow \quad b = \dfrac{19}{11}c$

On putting $b = \dfrac{19c}{11}$ in Eq. (ii), we get

$3a - \dfrac{19c}{11} + 2c = 0 \Rightarrow 3a + \dfrac{-19c + 22c}{11} = 0$

$\Rightarrow \quad 3a + \dfrac{3c}{11} = 0 \Rightarrow 3a = -\dfrac{3c}{11}$

$\therefore \quad a = -\dfrac{c}{11}$ **(1)**

Now, putting $a = -\dfrac{c}{11}$ and $b = \dfrac{19}{11}c$ in Eq. (i), we get the required equation of plane as

$\dfrac{-c}{11}x + \dfrac{19c}{11}y + cz = 0$

$\Rightarrow -\dfrac{x}{11} + \dfrac{19y}{11} + z = 0$

[dividing both sides by c]

$\Rightarrow \quad -x + 19y + 11z = 0$

[multiplying both sides by 11]

$\Rightarrow \quad x - 19y - 11z = 0$ **(1)**

17. Find the equation of plane that contains the point (1, –1, 2) and is perpendicular to each of planes $2x + 3y - 2z = 5$ and $x + 2y - 3z = 8$.

Delhi 2009C

The equation of any plane passing through (x_1, y_1, z_1) is

$$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$$

and if it is perpendicular to the planes

$a_1x+b_1y+c_1z+d_1=0$ and

$a_2x+b_2y+c_2z+d_2=0$.

Then, $aa_1+bb_1+cc_1=0$

and $aa_2+bb_2+cc_2=0$.

Use these results and solve it.

Equation of plane passing through point (1, −1, 2) is given by

$$a(x-1)+b(y+1)+c(z-2)=0 \quad \text{...(i)} \ \mathbf{(1)}$$

Now, given that plane (i) is perpendicular to planes

$$2x+3y-2z=5 \quad \text{...(ii)}$$

and $$x+2y-3z=8 \quad \text{...(iii)}$$

We know that, when two planes

$a_1x+b_1y+c_1z=d_1$ and $a_2x+b_2y+c_2z=d_2$ are perpendicular, then

$$a_1a_2+b_1b_2+c_1c_2=0 \quad \mathbf{(1)}$$

$\therefore$ $2a+3b-2c=0$ [from Eqs. (i) and (ii)]

and $a+2b-3c=0$ [from Eqs. (i) and (iii)]

$\Rightarrow$ $$2a+3b=2c \quad \text{...(iv)}$$

and $$a+2b=3c \quad \text{...(v)}\mathbf{(1)}$$

On multiplying Eq. (v) by 2 and subtracting it from Eq. (iv), we get

$$\begin{array}{r} 2a+3b = 2c \\ \underline{\overset{-}{}2a \pm 4b = \overset{}{-}6c} \\ -b = -4c \end{array}$$

$\Rightarrow$ $$b=4c$$

On putting $b=4c$ in Eq. (v), we get

$$a+8c=3c$$

$\Rightarrow$ $$a=-5c$$

Now, on putting $a = -5c$ and $b = 4c$ in Eq. (i), we get the required equation of plane as

$$-5c(x-1) + 4c(y+1) + c(z-2) = 0$$

$$\Rightarrow \quad -5(x-1) + 4(y+1) + (z-2) = 0$$

[dividing both sides by c]

$$\Rightarrow \quad -5x + 5 + 4y + 4 + z - 2 = 0$$

$$\Rightarrow \quad 5x - 4y - z - 7 = 0 \qquad \textbf{(1)}$$

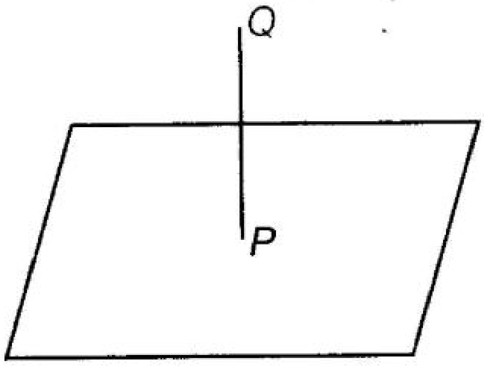
18. Find the coordinates of point, where the line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z+3}{4}$ meets the plane $x + y + 4z = 6$. **All India 2008**

Given equation of line is

$$\frac{x+1}{2}=\frac{y+2}{3}=\frac{z+3}{4},$$

then $\frac{x+1}{2}=\frac{y+2}{3}=\frac{z+3}{4}=\lambda$ [say] **(1)**

$\Rightarrow$ $x=2\lambda-1, y=3\lambda-2, z=4\lambda-3$



$\therefore$ Any point on the line is

$P(2\lambda-1, 3\lambda-2, 4\lambda-3)$ **(1)**

Now, as point P lies on the plane. So, it will satisfy the given equation of plane which is

$$x+y+4z=6$$

$\therefore$ $(2\lambda-1)+(3\lambda-2)+4(4\lambda-3)=6$ **(1)**

$\Rightarrow$ $2\lambda-1+3\lambda-2+16\lambda-12=6$

$\Rightarrow$ $21\lambda-21=0 \Rightarrow 21\lambda=21$

$\Rightarrow$ $\lambda=1$

On putting $\lambda=1$ in point P, we get the required point $P(2-1, 3-2, 4-3)=P(1,1,1)$.

(1)

6 Marks Questions

19. Find the equation of the plane passing through the line of intersection of the plane $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=1$ and $\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4=0$ and parallel to X-axis. **All India 2014C, 2011**

Given equation of planes are

$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ and $\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$

Above equations can be written in cartesian form as

$$x + y + z - 1 = 0 \quad \ldots\text{(i)}$$

and $$2x + 3y - z + 4 = 0 \quad \ldots\text{(ii)}$$ **(1)**

Let the required equation of plane passing through the line of intersection of planes (i) and (ii) be

$(x + y + z - 1) + \lambda\,(2x + 3y - z + 4) = 0$

$\Rightarrow x + y + z - 1 + 2\lambda x + 3\lambda y - \lambda z + 4\lambda = 0$

$\Rightarrow x\,(1 + 2\lambda) + y\,(1 + 3\lambda) + z\,(1 - \lambda)$

$+ (-1 + 4\lambda) = 0 \ldots\text{(iii)}$ **(1)**

$\therefore$ DR's of the above planes are $1+2\lambda, 1+3\lambda, 1-\lambda$.

Also, DR's of the X-axis are (1, 0, 0). **(1)**

Also, given that the above plane (iii) is parallel to the X-axis.

$$\therefore \quad 1(1+2\lambda)+0\,(1+3\lambda)+0\,(1-\lambda)=0 \quad \textbf{(1)}$$

$$\left[\begin{array}{ll} \because & a_1a_2+b_1b_2+c_1c_2=0 \\ \text{where,} & a_1=1+2\lambda,\ b_1=1+3\lambda,\ c_1=1-\lambda \\ \text{and} & a_2=1,\ b_2=0,\ c_2=0 \end{array}\right]$$

$$\Rightarrow \quad 1+2\lambda=0 \Rightarrow 2\lambda=-1$$

$$\Rightarrow \quad \lambda=-\frac{1}{2} \quad \textbf{(1)}$$

On putting $\lambda=-1/2$ in Eq. (iii), we get the required equation of plane as

$$x\left(1-\frac{2\times 1}{2}\right)+y\left(1-\frac{3}{2}\right)+z\left(1+\frac{1}{2}\right)+\left(-1-\frac{4}{2}\right)=0$$

$$\Rightarrow \frac{-y}{2}+\frac{3z}{2}-\frac{6}{2}=0 \Rightarrow y-3z+6=0$$

$\therefore$ The vector equation of plane is

$$\vec{r}\cdot(\hat{j}-3\hat{k})+6=0. \quad \textbf{(1)}$$

20. Find the distance between the point (7, 2, 4) and the plane determined by the points $A(2, 5, -3)$, $B(-2, -3, 5)$ and $C(5, 3, -3)$.

Delhi 2014

Given points are $A(2, 5, -3)$, $B(-2, -3, 5)$ and $C(5, 3, -3)$.

Consider $(x_1, y_1, z_1) = A(2, 5, -3)$

$(x_2, y_2, z_2) = B(-2, -3, 5)$

and $(x_3, y_3, z_3) = C(5, 3, -3)$ **(1)**

Now, equation of plane passing through three collinear points is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$ **(1)**

On putting the values of three points , we get

$$\begin{vmatrix} x - 2 & y - 5 & z + 3 \\ -2 - 2 & -3 - 5 & 5 + 3 \\ 5 - 2 & 3 - 5 & -3 + 3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x - 2 & y - 5 & z + 3 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 0$$ **(1)**

$$\Rightarrow (x - 2)(0 + 16) - (y - 5)(0 - 24) + (z + 3)(8 + 24) = 0$$

$\Rightarrow 16x - 32 + 24y - 120 + 32z + 96 = 0$

$\Rightarrow \quad 16x + 24y + 32z - 56 = 0$

$\Rightarrow \quad 2x + 3y + 4z - 7 = 0 \quad \ldots\text{(i)}$ **(1)**

Now, distance between the plane (i) and the point (7, 2, 4) is

$$d = \left| \frac{2(7) + 3(2) + 4(4) - 7}{\sqrt{(2)^2 + (3)^2 + (4)^2}} \right| \qquad \textbf{(1)}$$

$\left[\because \text{distance between the plane } ax + by + cz + d = 0 \text{ and the point } (x_1 y_1 z_1) \text{ is } \left| \frac{ax_1 + by_1 + cz_1 - d}{\sqrt{a^2 + b^2 + c^2}} \right| \right]$

$$= \left| \frac{14 + 6 + 16 - 7}{\sqrt{29}} \right|$$

$$= \frac{29}{\sqrt{29}}$$

$$= \sqrt{29} \text{ units} \qquad \textbf{(1)}$$

21. Find the equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$, which is perpendicular to the plane $x - y + z = 0$. Also, find the distance of the plane obtained above, from the origin. **All India 2014**

Equation of any plane through the line of intersection of the given planes $x + y + z = 1$ and $2x + 3y + 4z = 5$, is

$$(x + y + z - 1) + \lambda (2x + 3y + 4z - 5) = 0$$

$$\Rightarrow \quad (1 + 2\lambda) x + (1 + 3\lambda) y + (1 + 4\lambda) z - 1 - 5\lambda = 0 \quad \text{...(i)} \quad \textbf{(1)}$$

The direction ratios a_1, b_1, c_1 of the plane are $(2\lambda + 1), (3\lambda + 1)$ and $(4\lambda + 1)$.

Also, given that the plane, i.e. Eq. (i) is perpendicular to the plane $x - y + z = 0$, whose direction ratios a_2, b_2, c_2 are $1, -1$ and 1. **(1)**

Then, $\quad a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$\Rightarrow \quad 1(1 + 2\lambda) - 1(1 + 3\lambda) + 1(1 + 4\lambda) = 0$$

$$\Rightarrow \quad 1 + 2\lambda - 1 - 3\lambda + 1 + 4\lambda = 0$$

$$\Rightarrow \quad 3\lambda = -1 \Rightarrow \lambda = -\frac{1}{3} \quad \textbf{(1)}$$

On substituting the value of λ in Eq. (i), we get the equation required plane as

$$\left(1 - \frac{2}{3}\right) x + \left(1 - \frac{3}{3}\right) y + \left(1 - \frac{4}{3}\right) z - 1 + \frac{5}{3} = 0$$

$$\Rightarrow \quad \frac{1}{3} x - \frac{1}{3} z + \frac{2}{3} = 0$$

$$\Rightarrow \quad x - z + 2 = 0 \quad \textbf{(1)}$$

Now, we know that, distance between a point $P(x_1, y_1, z_1)$ and plane $Ax + By + Cz = D$ is

$$d = \frac{Ax_1 + By_1 + Cz_1 - D}{\sqrt{A^2 + B^2 + C^2}} \quad \textbf{(1)}$$

Here, point is $(0, 0, 0)$ and the plane is $x - z + 2 = 0$.

$\therefore$ Required distance,

$$d = \left| \frac{1 \times 0 + 0 + (-1) \times 0 + 2}{\sqrt{(1)^2 + (-1)^2}} \right| = \frac{2}{\sqrt{2}} = \sqrt{2} \text{ unit}$$

(1)

22. Find the distance of the point $(2, 12, 5)$ from the point of intersection of the line $\vec{r} = 2\hat{i} - 4\hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 0$. **All India 2014**

Given equation of line is

$$\vec{r} = 2\hat{i} - 4\hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k}) \quad ...(i)$$

and the equation of plane is

$$\vec{r} \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 0 \quad ...(ii)$$

(1)

For point of intersection of Eqs. (i) and (ii), we get

$$[2\hat{i} - 4\hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})] \cdot [\hat{i} - 2\hat{j} + \hat{k}] = 0 \ \textbf{(1)}$$

[on putting the value of $\vec{r}$ from Eq. (i) to Eq. (ii)]

$$\Rightarrow \quad 2 + 8 + 2 + 3\lambda - 8\lambda + 2\lambda = 0$$

$$\Rightarrow \quad 12 - 3\lambda = 0$$

$$\Rightarrow \quad -3\lambda = -12 \Rightarrow \lambda = 4 \quad \textbf{(1)}$$

On putting $\lambda = 4$ in Eq. (i), we get

$$\vec{r} = 2\hat{i} - 4\hat{j} + 2\hat{k} + 4(3\hat{i} + 4\hat{j} + 2\hat{k})$$

$$= 14\hat{i} + 12\hat{j} + 10\hat{k} \quad \textbf{(1)}$$

Since, $\vec{r}$ is the position vector of the point $(14, 12, 10)$. **(1)**

$\therefore$ Distance between the points $(2, 12, 5)$ and $(14, 12, 10)$

$$= \sqrt{(2-14)^2 + (12-12)^2 + (5-10)^2}$$

$$= \sqrt{(-12)^2 + (0)^2 + (-5)^2}$$

$$= \sqrt{144 + 25}$$

$$= \sqrt{169}$$

$$= 13 \text{ units} \quad \textbf{(1)}$$

23. Find the equation of the plane that contains the point (1, –1, 2) and is perpendicular to both the planes $2x+3y-2z=5$ and $x+2y-3z=8$. Hence, find the distance of point $P(-2, 5, 5)$ from the plane obtained above. **Foreign 2014**

Given point is (1, –1, 2) whose position vector is $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$.

Also, given equation of planes are

$2x + 3y - 2z$ and $x + 2y - 3z = 8$ **(1)**

Vector forms of these planes are

$\vec{N_1} = 2\hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{N_2} = \hat{i} + 2\hat{j} - 3\hat{k}$. **(1)**

Now, required planes is prependicular to given planes, so the normal vector of the required plane

$$\vec{N} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -2 \\ 1 & 2 & -3 \end{vmatrix}$$

$$= \hat{i}(-9+4) - \hat{j}(-6+2) + \hat{k}(4-3)$$

$$= -5\hat{i} + 4\hat{j} + \hat{k}$$ **(1)**

So, the equation of the required plane is

$(\vec{r} - \vec{a}) \cdot \vec{N} = 0$

$\Rightarrow [\vec{r} - (\hat{i} - \hat{j} + 2\hat{k})] \cdot (-5\hat{i} + 4\hat{j} + \hat{k}) = 0$ **(1)**

$\Rightarrow \vec{r}(-5\hat{i} + 4\hat{j} + \hat{k}) + 7 = 0$ is the vector equation and $5x - 4y - z = 7$ is the cartesian equation of required plane. **(1)**

Also, the distance of point $P(-2, 5, 5)$ from the plane obtained $= \left|\dfrac{5(-2) - 4(5) - (5) - 7}{\sqrt{25+16+1}}\right|$

$$= \left|\frac{-42}{\sqrt{42}}\right| = \sqrt{42} \text{ units}$$ **(1)**

24. Show that the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ are coplanar. Also, find the equation of the plane containing them. **Delhi 2013C**

Given lines are $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$

and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$

On comparing both equations of lines with $\vec{r} = \vec{a} + \lambda\vec{b}$ respectively, we get

$$\vec{a_1} = \hat{i} + \hat{j} - \hat{k}, \vec{b_1} = 3\hat{i} - \hat{j}$$

and $\vec{a_2} = 4\hat{i} - \hat{k}, \vec{b_2} = 2\hat{i} + 3\hat{k}$ **(1)**

then, $\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix}$

$$= \hat{i}(-3 - 0) - \hat{j}(9 - 0) + \hat{k}(0 + 2)$$

$$= -3\hat{i} - 9\hat{j} + 2\hat{k}$$ **(1)**

and $\vec{a_2} - \vec{a_1} = (4\hat{i} - \hat{k}) - (\hat{i} + \hat{j} - \hat{k}) = 3\hat{i} - \hat{j}$ **(1)**

Now, $(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = (3\hat{i} - \hat{j})$

$$\cdot(-3\hat{i} - 9\hat{j} + 2\hat{k})$$

$$= -9 + 9 = 0$$

Hence, given lines are coplanar. **(1)**

Now, cartesian equations of given lines are

$$\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0} \text{ and}$$

$$\frac{x-4}{2} = \frac{y-0}{0} = \frac{z+1}{3},$$

Then, equation of plane containing them is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-1 & y-1 & z+1 \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix} = 0$$ **(1)**

$\Rightarrow (x-1)(-3-0) - (y-1)(9-0) + (z+1)(0+2) = 0$

$\Rightarrow -3x + 3 - 9y + 9 + 2z + 2 = 0$

$\therefore \quad 3x + 9y - 2z = 14$ **(1)**

25. Find the distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to the line $\frac{x-1}{2} = \frac{y-3}{3} = \frac{z+2}{-6}$. **Delhi 2013C**

Let $P(1, -2, 3)$ be the given point.

Given equation of line is

$$\frac{x-1}{2} = \frac{y-3}{3} = \frac{z+2}{-6} = \lambda \qquad \text{[say]}$$

$\Rightarrow \quad x = 2\lambda + 1, y = 3\lambda + 3, z = -6\lambda - 2$

Any point on the line is $Q(2\lambda + 1, 3\lambda + 3, -6\lambda - 2)$.

Now, direction ratios of PQ are

$(2\lambda + 1 - 1, 3\lambda + 3 + 2, -6\lambda - 2 - 3)$

i.e. $\quad (2\lambda, 3\lambda + 5, -6\lambda - 5)$ **(1)**

According to question, the line PQ is parallel to the plane

$$x - y + z = 5$$

Therefore, normal to the plane is perpendicular to the line.

i.e. $\quad a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

$\therefore \quad 2\lambda(1) + (3\lambda + 5)(-1) + (-6\lambda - 5)(1) = 0$ **(1)**

$\Rightarrow \quad 2\lambda - 3\lambda - 5 - 6\lambda - 5 = 0$

$\Rightarrow \quad -7\lambda - 10 = 0 \Rightarrow \lambda = \dfrac{-10}{7}$ **(1)**

$\therefore$ Coordinates of Q are

$$Q\left[2\times\left(\frac{-10}{7}\right)+1, 3\left(\frac{-10}{7}\right)+3, -6\left(\frac{-10}{7}\right)-2\right]$$

i.e. $$Q\left(\frac{-13}{7}, \frac{-9}{7}, \frac{46}{7}\right)$$ **(1)**

Hence, distance between the points

$P(1, -2, 3)$ and $Q\left(\frac{-13}{7}, \frac{-9}{7}, \frac{46}{7}\right)$

$$=\sqrt{\left(\frac{-13}{7}-1\right)^2+\left(\frac{-9}{7}+2\right)^2+\left(\frac{46}{7}-3\right)^2}$$ **(1)**

$$\left[\because \text{distance} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}\right]$$

$$=\sqrt{\left(\frac{-20}{7}\right)^2+\left(\frac{5}{7}\right)^2+\left(\frac{25}{7}\right)^2}$$

$$=\frac{1}{7}\sqrt{400+25+625}$$

$$=\frac{\sqrt{1050}}{7}=\frac{5\sqrt{6\times 7}}{7}=5\sqrt{\frac{6}{7}} \text{ units}$$ **(1)**

26. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}\cdot(\hat{i}+3\hat{j})-6=0$ and $\vec{r}\cdot(3\hat{i}-\hat{j}-4\hat{k})=0$, whose perpendicular distance from origin is unity.

All India 2013C

Given planes are $\vec{r}\cdot(\hat{i}+3\hat{j})-6=0$ and $\vec{r}\cdot(3\hat{i}-\hat{j}-4\hat{k})=0$. The equation of any plane passing through the line of intersection of these planes is

$$[\vec{r}\cdot(\hat{i}+3\hat{j})-6]+\lambda[\vec{r}\cdot(3\hat{i}-\hat{j}-4\hat{k})]=0$$

$$\Rightarrow \vec{r}\cdot[(3\lambda+1)\hat{i}+(3-\lambda)\hat{j}-4\lambda\hat{k}]-6=0 \quad \text{...(i)} \quad \textbf{(1)}$$

Given, perpendicular distance of origin from this plane is unity.

$$\therefore \frac{|(3\lambda+1)\times 0+(3-\lambda)\times 0-4\lambda\times 0-6|}{\sqrt{(3\lambda+1)^2+(3-\lambda)^2+(-4\lambda)^2}}=1 \quad \textbf{(1)}$$

$$\Rightarrow \quad 6=\sqrt{9\lambda^2+1+6\lambda+9+\lambda^2-6\lambda+16\lambda^2}$$

$$\Rightarrow \quad 6=\sqrt{26\lambda^2+10} \quad \textbf{(1)}$$

On squaring both sides, we get

$$26\lambda^2+10=36 \Rightarrow 26\lambda^2=26$$

$$\Rightarrow \quad \lambda^2=1 \quad \Rightarrow \quad \lambda=\pm 1 \quad \textbf{(1)}$$

On putting $\lambda=1$ in Eq. (i), we get the required equation of plane as

$$\vec{r}\cdot[(3+1)\hat{i}+(3-1)\hat{j}-4\hat{k}]-6=0$$

$$\Rightarrow \quad \vec{r}(4\hat{i}+2\hat{j}-4\hat{k})-6=0 \quad \textbf{(1)}$$

On putting $\lambda=-1$ in Eq. (i), we get the required equation of plane as

$$\vec{r}[(3(-1)+1)\hat{i}+(3+1)\hat{j}-4(-1)\hat{k}]-6=0$$

$$\Rightarrow \quad \vec{r}[-2\hat{i}+4\hat{j}+4\hat{k}]-6=0 \quad \textbf{(1)}$$

27. Find the vector equation of the line passing through the point (1, 2, 3) and parallel to the planes $\vec{r}\cdot(\hat{i}-\hat{j}+2\hat{k})=5$ and $\vec{r}\cdot(3\hat{i}+\hat{j}+\hat{k})=6$.

All India 2013

Suppose the required line is parallel to vector $\vec{b}$ which is given by $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$.

The position vector of the point (1, 2, 3) is

$$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$$

The equation of line passing through (1, 2, 3) and parallel to $\vec{b}$ is given by

$$\vec{r} = \vec{a} + \lambda\,\vec{b}$$

$$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda\,(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) \quad \text{...(i)}$$

(1)

The equation of the given planes are

$$\vec{r}\cdot(\hat{i} - \hat{j} + 2\hat{k}) = 5 \quad \text{...(ii)}$$

and $$\vec{r}\cdot(3\hat{i} + \hat{j} + \hat{k}) = 6 \quad \text{...(iii)}$$ **(1)**

The line in Eq. (i) and plane in Eq. (ii) are parallel.

Therefore, the normal to the plane of Eq. (ii) and the given line are perpendicular.

$\therefore\ (\hat{i} - \hat{j} + 2\hat{k})\cdot\lambda\,(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) = 0$ **(1)**

$$\Rightarrow \lambda(b_1 - b_2 + 2b_3) = 0$$

$$\Rightarrow b_1 - b_2 + 2b_3 = 0 \quad \text{...(iv)}$$

Similarly, from Eqs. (i) and (ii), we get

$$(3\hat{i} + \hat{j} + \hat{k})\cdot\lambda\,(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) = 0$$

$$\Rightarrow \lambda(3b_1 + b_2 + b_3) = 0$$

$$\Rightarrow 3b_1 + b_2 + b_3 = 0 \quad \text{...(v)}$$ **(1)**

From Eqs. (iv) and (v), we get

b_1 b_2 b_3

$$\frac{\quad}{(-1)\times 1-1\times 2}=\frac{\quad}{2\times 3-1\times 1}=\frac{\quad}{1\times 1-3(-1)}$$

$$\Rightarrow \qquad \frac{b_1}{-3}=\frac{b_2}{5}=\frac{b_3}{4} \qquad \textbf{(1)}$$

Therefore, the direction ratios of $\vec{b}$ are $-3, 5$ and 4.

$\therefore \ \vec{b} = -3\hat{i} + 5\hat{j} + 4\hat{k}$ $[\because \ \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}]$

On substituting the value of $\vec{b}$ in Eq. (i), we get

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k}) \qquad \textbf{(1)}$$

which is the equation of the required line.

28. Find the coordinates of the point, where the line through $(3,-4,-5)$ and $(2,-3,1)$ crosses the plane, passing through the point $(2,2,1)$, $(3,0,1)$ and $(4,-1,0)$. **Delhi 2013**

Equation of the line passing through the points $(3,-4,-5)$ and $(2,-3,1)$ is

$$\frac{x-3}{2-3}=\frac{y+4}{-3+4}=\frac{z+5}{1+5}$$

$$\left[\because \frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}\right]$$

$$\Rightarrow \qquad \frac{x-3}{-1}=\frac{y+4}{1}=\frac{z+5}{6}=\lambda \qquad \text{[say]} \ \textbf{(1½)}$$

$\Rightarrow \quad x=-\lambda+3, y=\lambda-4, z=6\lambda-5$

$\therefore$ Any point on this line be

$$(-\lambda+3, \lambda-4, 6\lambda-5). \qquad \textbf{(1/2)}$$

Now, equation of plane passes through the points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} \qquad \textbf{(1)}$$

$\therefore$ Equation of plane passes through the points $(2, 2, 1)$, $(3, 0, 1)$ and $(4, -1, 0)$ is

$$\begin{vmatrix} x-2 & y-2 & z-1 \\ 3-2 & 0-2 & 1-1 \\ 4-2 & -1-2 & 0-1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-2 & y-2 & z-1 \\ 1 & -2 & 0 \\ 2 & -3 & -1 \end{vmatrix} = 0$$

$$\Rightarrow (x-2)(2-0)-(y-2)(-1-0) + (z-1)(-3+4)=0$$

$$\Rightarrow 2x-4+y-2+z-1=0$$

$$\Rightarrow 2x+y+z-7=0 \quad \text{...(i)} \quad \textbf{(1)}$$

Also, the point of line lies on plane (i).

$$\therefore \quad 2(-\lambda+3)+(\lambda-4)+(6\lambda-5)-7=0$$

$$\Rightarrow -2\lambda+6+\lambda-4+6\lambda-5-7=0$$

$$\Rightarrow 5\lambda-10=0$$

$$\Rightarrow \lambda=2 \quad \textbf{(1)}$$

Hence, point of intersection of line and plane is

$(-2+3, 2-4, 12-5)$, i.e. $(1, -2, 7)$. **(1)**

29. Find the vector equation of the plane passing through the three points with position vectors $\hat{i}+\hat{j}-2\hat{k}$, $2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$. Also, find the coordinates of the point of intersection of this plane and the line $\vec{r}=3\hat{i}-\hat{j}-\hat{k}+\lambda(2\hat{i}-2\hat{j}+\hat{k})$. **Delhi 2013**

Given position vectors of three points are $(\hat{i}+\hat{j}-2\hat{k}), 2\hat{i}-\hat{j}+\hat{k}$ and $(\hat{i}+2\hat{j}+\hat{k})$.

We know that, the vector equation of the plane passing through the three points is

$$(\vec{r}-\vec{a})\cdot(\vec{b}\times\vec{c})=0 \quad \textbf{(1)}$$

Now, $$\vec{b}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 2 & 1 \end{vmatrix} = -3\hat{i}-\hat{j}+5\hat{k} \quad \textbf{(1)}$$

$\therefore$ Equation of plane is

$$[\vec{r} - (\hat{i} + \hat{j} - 2\hat{k})] \cdot (-3\hat{i} - \hat{j} + 5\hat{k}) = 0$$

$$\Rightarrow \quad \vec{r} \cdot (-3\hat{i} - \hat{j} + 5\hat{k}) = -3 - 1 - 10$$

$$\Rightarrow \quad \vec{r} \cdot (-3\hat{i} - \hat{j} + 5\hat{k}) = -14$$

$$\Rightarrow \quad \vec{r} \cdot (3\hat{i} + \hat{j} - 5\hat{k}) = 14 \qquad \text{...(i)} \ \mathbf{(1½)}$$

Also, given equation of line is

$$\vec{r} = 3\hat{i} - \hat{j} - \hat{k} + \lambda(2\hat{i} - 2\hat{j} + \hat{k}) \qquad \text{...(ii)}$$

This line intersect the plane (i), so

$$[(3 + 2\lambda)\hat{i} + (-1 - 2\lambda)\hat{j} + (-1 + \lambda)\hat{k}] \cdot (3\hat{i} + \hat{j} - 5\hat{k}) = 14$$

$$\Rightarrow \quad 3 \cdot (3 + 2\lambda) + 1 \cdot (-1 - 2\lambda) - 5(-1 + \lambda) = 14$$

$$\Rightarrow \quad 9 + 6\lambda - 1 - 2\lambda + 5 - 5\lambda = 14$$

$$\Rightarrow \quad -\lambda = 14 - 13$$

$$\Rightarrow \quad -\lambda = 1$$

$$\therefore \quad \lambda = -1 \qquad \mathbf{(1½)}$$

On putting $\lambda = -1$ in Eq. (ii), the required point of intersection is

$$\vec{r} = \hat{i} + \hat{j} - 2\hat{k} \qquad \mathbf{(1)}$$

30. Find the equation of plane determined by points $A(3, -1, 2)$, $B(5, 2, 4)$ and $C(-1, -1, 6)$ and hence find the distance between planes and point $(6, 5, 9)$.

Delhi 2013, 2012; All India 2009, 2008C

Given points are $A(3, -1, 2)$ $B(5, 2, 4)$, and $C(-1, -1, 6)$.

Now, $AB = \sqrt{(5-3)^2 + (2+1)^2 + (4-2)^2}$

$= \sqrt{4+9+4} = \sqrt{17}$ units

$BC = \sqrt{(-1-5)^2 + (-1-2)^2 + (6-4)^2}$

$= \sqrt{36+9+4}$

$= \sqrt{49} = 7$ units

and $CA = \sqrt{(3+1)^2 + (-1+1)^2 + (2-6)^2}$

$= \sqrt{16+0+16}$ **(1)**

$= \sqrt{32}$ units.

$\because AB + BC \neq CA$, so given points are non-collinear. Now, equation of plane passing through three non-collinear points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

Here, $(x_1, y_1, z_1) = (3, -1, 2)$,

$(x_2, y_2, z_2) = (5, 2, 4)$ **(1)**

and $(x_3, y_3, z_3) = (-1, -1, 6)$

Equation of plane is

$$\begin{vmatrix} x-3 & y+1 & z-2 \\ 5-3 & 2+1 & 4-2 \\ -1-3 & -1+1 & 6-2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-3 & y+1 & z-2 \\ 2 & 3 & 2 \\ -4 & 0 & 4 \end{vmatrix} = 0 \quad \textbf{(1)}$$

$\Rightarrow (x-3)[12-0] - (y+1)(8+8) + (z-2)(0+12) = 0$

$\Rightarrow 12(x-3) - 16(y+1) + 12(z-2) = 0$

$\Rightarrow \quad 12(x-3)-16(y+1)+12(z-2)=0$

$\Rightarrow \quad 12x-36-16y-16+12z-24=0$

$\Rightarrow \quad 12x-16y+12z-76=0$

On dividing both sides by 4, we get the required equation of plane as

$$3x-4y+3z-19=0 \qquad \text{...(i)} \ \mathbf{(1½)}$$

Now, distance of above plane (i) from point $P(6,5,9)$ is

$$d=\frac{|3(6)+(-4)(5)+(3)(9)-19|}{\sqrt{(3)^2+(-4)^2+(3)^2}}$$

$$\left[\because d=\frac{|Ax_1+By_1+Cz_1+D|}{\sqrt{A^2+B^2+C^2}}\right]$$

$$=\frac{|18-20+27-19|}{\sqrt{9+16+9}}$$

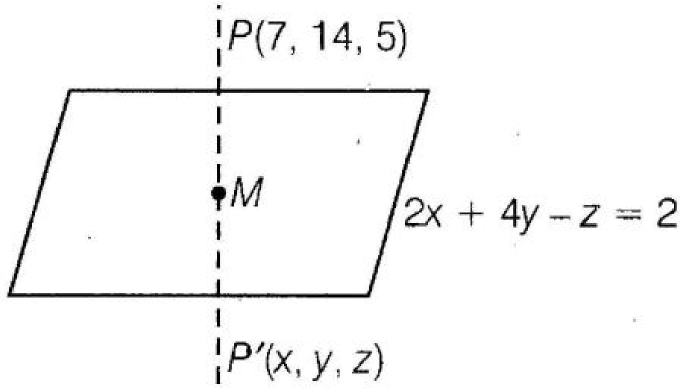
$$=\frac{6}{\sqrt{34}} \text{ units} \qquad \mathbf{(1½)}$$

31. Find the length and foot of perpendicular from point $P(7,14,5)$ to plane $2x+4y-z=2$. Also, find the image of point P in the plane.

HOTS; All India 2012

Let $P'(x,y,z)$ be the image of the given point $P(7,14,5)$ and M be the foot of perpendicular lie on the plane $2x+4y-z=2$. The equation of line PM in plane is given by

$$\frac{x-7}{2}=\frac{y-14}{4}=\frac{z-5}{-1}$$



[∵ DR's of a line is proportional to the normal to the plane] **(1)**

Let $\frac{x-7}{2}=\frac{y-14}{4}=\frac{z-5}{-1}=\lambda$ [say]

$\Rightarrow x = 2\lambda + 7, y = 4\lambda + 14$ and $z = -\lambda + 5$

Let coordinates of point M be

$$(2\lambda + 7, 4\lambda + 14, -\lambda + 5) \quad \text{...(i)}$$ **(1)**

Since, M lies on the given plane $2x + 4y - z = 2$.

$\therefore \quad 2(2\lambda + 7) + 4(4\lambda + 14) - (-\lambda + 5) = 2$

$\Rightarrow \quad 4\lambda + 14 + 16\lambda + 56 + \lambda - 5 = 2$

$\Rightarrow \quad 21\lambda + 63 = 0$

$\Rightarrow \quad \lambda = \frac{-63}{21} = -3$

(1)

On putting $\lambda = -3$ in Eq. (i), we get

$$M = (1, 2, 8)$$

$\therefore$ Foot of perpendicular M is $(1, 2, 8)$. **(1)**

Also, length of perpendicular PM = Distance between points P and M

$$= \sqrt{(1-7)^2 + (2-14)^2 + (8-5)^2}$$

$$\left[\because \text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}\right]$$

$$= \sqrt{36 + 144 + 9}$$

$$= \sqrt{189} \text{ units}$$ **(1)**

Now, $\quad M$ = Mid-point of P and P'

$$\Rightarrow (1, 2, 8) = \left(\frac{x+7}{2}, \frac{y+14}{2}, \frac{z+5}{2}\right)$$

On equating corresponding coordinates, we get

$$\frac{x+7}{2}=1, \frac{y+14}{2}=2 \text{ and } \frac{z+5}{2}=8$$

$\Rightarrow \quad x=2-7, y=4-14$ and $z=16-5$

$\Rightarrow \quad x=-5, y=-10$ and $z=11$

Hence, image of point $P(7,14,5)$ is

$P'(-5,-10,11)$. **(1)**

32. Find the equation of plane which contains the line of intersection of planes $\vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k})-4=0$, $\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})+5=0$ and which is perpendicular to plane $\vec{r}\cdot(5\hat{i}+3\hat{j}-6\hat{k})+8=0$. **All India 2011**

Given, the required plane contains the line of intersection of planes whose equations are

$$\vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k})-4=0 \quad \text{...(i)}$$

and $$\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})+5=0 \quad \text{...(ii)}$$

Eqs. (i) and (ii) can be written in cartesian form

$$x+2y+3z-4=0$$

and $$2x+y-z+5=0$$ **(1)**

So, the required equation of plane is

$(x+2y+3z-4)+\lambda \quad (2x+y-z+5)=0$ ...(iii)

$\Rightarrow x+2y+3z-4+2\lambda x+\lambda y - \lambda z+5\lambda=0$ **(1/2)**

$\Rightarrow x(1+2\lambda)+y(2+\lambda)+z(3-\lambda) + (-4+5\lambda)=0$ ...(iv) **(1/2)**

Also, given that plane in Eq. (iv) is perpendicular to the plane

$$\vec{r}\cdot(5\hat{i}+3\hat{j}-6\hat{k})+8=0$$

which in cartesian form can be written as

$$5x + 3y - 6z + 8 = 0 \quad \ldots(v)$$

$\therefore \quad 5(1 + 2\lambda) + 3(2 + \lambda) - 6(3 - \lambda) = 0$

$$\left[\begin{array}{l} \because a_1a_2 + b_1b_2 + c_1c_2 = 0, \\ \text{where } a_1 = 1 + 2\lambda, b_1 = 2 + \lambda, c_1 = 3 - \lambda \\ \text{and } a_2 = 5, b_2 = 3, c_2 = -6 \end{array}\right]$$

(1)

$\Rightarrow \quad 5 + 10\lambda + 6 + 3\lambda - 18 + 6\lambda = 0$

$\Rightarrow \quad 19\lambda - 7 = 0 \Rightarrow \lambda = \dfrac{7}{19}$ **(1)**

On putting $\lambda = \dfrac{7}{19}$ in Eq. (iii), we get the required equation of plane as

$$(x + 2y + 3z - 4) + \frac{7}{19}(2x + y - z + 5) = 0$$

$\Rightarrow \quad 19x + 38y + 57z - 76 + 14x + 7y - 7z + 35 = 0$ **(1)**

$\Rightarrow \quad 33x + 45y + 50z - 41 = 0$

In vector form, the required equation of plane is $\vec{r} \cdot (33\hat{i} + 45\hat{j} + 50\hat{k}) = 41.$ **(1)**

33. Find the equation of plane passing through the line of intersection of planes $2x + y - z = 3$ and $5x - 3y + 4z + 9 = 0$ and parallel to line $\dfrac{x-1}{2} = \dfrac{y-3}{4} = \dfrac{z-5}{5}$.

HOTS; All India 2011

Given equations of planes are

$$2x + y - z - 3 = 0 \quad \text{...(i)}$$

and $$5x - 3y + 4z + 9 = 0 \quad \text{...(ii)} \; \textbf{(1)}$$

Let the required equation of plane which passes through the line of intersection of planes (i) and (ii) be

$$(2x + y - z - 3) + \lambda (5x - 3y + 4z + 9) = 0 \quad \text{...(iii)}$$

$$\Rightarrow x(2 + 5\lambda) + y(1 - 3\lambda) + z(-1 + 4\lambda) + (-3 + 9\lambda) = 0 \quad \text{...(iv)} \; \textbf{(1)}$$

Here, DR's of plane are $2 + 5\lambda, 1 - 3\lambda, -1 + 4\lambda$. Also, given that the plane (i) is parallel to the line, whose equation is

$$\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{5}$$

DR's of the line are 2, 4, 5.

Since, the plane is parallel to the line.

Therefore, normal to the plane is perpendicular to the line.

i.e. $a_1a_2 + b_1b_2 + c_1c_2 = 0$ **(1)**

Here, $a_1 = 2 + 5\lambda, b_1 = 1 - 3\lambda, c_1 = -1 + 4\lambda$

and $a_2 = 2, b_2 = 4, c_2 = 5$

$\therefore \; 2(2 + 5\lambda) + 4(1 - 3\lambda) + 5(-1 + 4\lambda) = 0$

$\Rightarrow \; 4 + 10\lambda + 4 - 12\lambda - 5 + 20\lambda = 0$

$\Rightarrow \; 18\lambda + 3 = 0 \Rightarrow \lambda = -\frac{3}{18} = -\frac{1}{6}$ **(1½)**

On putting $\lambda = -\frac{1}{6}$ in Eq. (iii), we get the required equation of plane as

$$(2x + y - z - 3) - \frac{1}{6}(5x - 3y + 4z + 9) = 0$$

$\Rightarrow 12x + 6y - 6z - 18 - 5x + 3y - 4z - 9 = 0$

$\therefore \quad 7x + 9y - 10z - 27 = 0$ **(1½)**

34. Find the equation of plane passing through the point (–1, 3, 2) and perpendicular to each of the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 5$. **Foreign 2011; All India 2009**

Let the required equation of plane passing through (–1, 3,2) be

$a(x+1) + b(y-3) + c(z-2) = 0$ ...(i) **(1)**

$\left[\begin{array}{l} \because \text{ equation of plane passing through} \\ (x_1, y_1, z_1) \text{ is} \\ a(x-x_1) + b(y-y_1) + c(z-z_1) = 0 \end{array}\right]$

Given that plane (i) is perpendicular to the planes whose equations are

$$x + 2y + 3z = 5 \quad \text{...(ii)}$$

and $$3x + 3y + z = 5 \quad \text{...(iii)}$$

$\therefore \quad a_1a_2 + b_1b_2 + c_1c_2 = 0$ **(1)**

Using the above result first in Eqs. (i), (ii) and then in Eqs. (i), (iii), we get

$$a + 2b + 3c = 0 \quad \text{...(iv)}$$

and $$3a + 3b + c = 0 \quad \text{...(v)} \textbf{(1)}$$

On multiplying Eq. (iv) by 3 and subtracting it from Eq. (v), we get

$$\begin{array}{r} 3a + 3b + c = 0 \\ 3a + 6b + 9c = 0 \\ - \quad - \quad - \quad \\ \hline -3b - 8c = 0 \end{array}$$

$\Rightarrow \quad -3b = 8c$

$\Rightarrow \quad b = -\frac{8c}{3}$

On putting $b = -\frac{8c}{3}$ in Eq. (iv), we get

$$a + 2\left(-\frac{8c}{3}\right) + 3c = 0$$

$\Rightarrow \quad a - \frac{16c}{3} + 3c = 0$

$$\Rightarrow \quad a = \frac{16c}{3} - 3c$$

$$= \frac{16c - 9c}{3}$$

$$= \frac{7c}{3}$$

$$\Rightarrow \quad a = \frac{7c}{3} \qquad \textbf{(1½)}$$

Now, on putting $a = \frac{7c}{3}$ and $b = -\frac{8c}{3}$ in Eq. (i), we get the required equation of plane as

$$\frac{7c}{3}(x+1) - \frac{8c}{3}(y-3) + c(z-2) = 0$$

On dividing both sides by c, we get

$$\frac{7}{3}(x+1) - \frac{8}{3}(y-3) + (z-2) = 0$$

$$\Rightarrow \quad 7x + 7 - 8y + 24 + 3z - 6 = 0$$

[multiplying both sides by 3]

$$\therefore \quad 7x - 8y + 3z + 25 = 0 \qquad \textbf{(1½)}$$

35. Find the vector equation of plane passing through the points $A(2, 2, -1)$, $B(3, 4, 2)$ and $C(7, 0, 6)$. Also, find the cartesian equation of plane. **Foreign 2011**

First, we check whether the points are collinear or not.

Given points are $A(2, 2, -1)$, $B(3, 4, 2)$ and $C(7, 0, 6)$.

$$\therefore \quad AB = \sqrt{(3-2)^2 + (4-2)^2 + (2+1)^2}$$

$$\left[\because \text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}\right]$$

$$= \sqrt{1 + 4 + 9} = \sqrt{14} \text{ units}$$

$$BC = \sqrt{(7-3)^2 + (0-4)^2 + (6-2)^2}$$
$$= \sqrt{16+16+16} = \sqrt{48} = 4\sqrt{3} \text{ units}$$
$$\text{and } CA = \sqrt{(2-7)^2 + (2-0)^2 + (-1-6)^2}$$
$$= \sqrt{25+4+49} = \sqrt{78} \text{ units}$$

$\because AB + BC \neq CA$, so points A, B, C are non-collinear. **(1)**

Now, the equation of plane passing through three non-collinear points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

Here, $(x_1, y_1, z_1) = (2, 2, -1)$, $(x_2, y_2, z_2) = (3, 4, 2)$ and $(x_3, y_3, z_3) = (7, 0, 6)$

$\therefore$ Equation of plane is

$$\begin{vmatrix} x-2 & y-2 & z+1 \\ 3-2 & 4-2 & 2+1 \\ 7-2 & 0-2 & 6+1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-2 & y-2 & z+1 \\ 1 & 2 & 3 \\ 5 & -2 & 7 \end{vmatrix} = 0$$ **(1½)**

Expanding along R_1, we get

$$(x-2)(14+6) - (y-2)(7-15) + (z+1)(-2-10) = 0$$

$$\Rightarrow (x-2)\cdot 20 - (y-2)(-8) + (z+1)(-12) = 0$$

$$\Rightarrow 20x - 40 + 8y - 16 - 12z - 12 = 0$$

$$\Rightarrow 20x + 8y - 12z - 68 = 0$$ **(1½)**

On dividing both sides by 4, we get

$$5x + 2y - 3z = 17$$

which is the required cartesian equation of plane.

Now, we know that, vector form of cartesian equation $ax + by + cz = d$ of plane is given by

$$\vec{r} \cdot (a\hat{i} + b\hat{j} + c\hat{k}) = d \quad \textbf{(1)}$$

$\therefore$ Required vector equation of plane is

$$\vec{r} \cdot (5\hat{i} + 2\hat{j} - 3\hat{k}) = 17 \quad \textbf{(1)}$$

36. Find the equation of plane passing through point $(1, 1, -1)$ and perpendicular to planes $x + 2y + 3z - 7 = 0$ and $2x - 3y + 4z = 0$.

Foreign 2011

Do same as Que. 34.

[**Ans.** $7x + 2y - 7z - 26 = 0$]

37. Find the vector and cartesian equation of a plane containing the two lines

$\vec{r} = (2\hat{i} + \hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 5\hat{k})$ and

$\vec{r} = (3\hat{i} + 3\hat{j} + 2\hat{k}) + \mu(3\hat{i} - 2\hat{j} + 5\hat{k})$.

Also, show that the line

$\vec{r} = (2\hat{i} + 5\hat{j} + 2\hat{k}) + P(3\hat{i} - 2\hat{j} + 5\hat{k})$ lies in the plane. **All India 2011C**

Given equations of lines are

$$\vec{r} = (2\hat{i} + \hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 5\hat{k}) \quad \ldots(i)$$

and $\vec{r} = (3\hat{i} + 3\hat{j} + 2\hat{k}) + \mu(3\hat{\imath} - 2\hat{j} + 5\hat{k})$...(ii)**(1)**

On comparing Eqs. (i) and (ii) with the vector equation of line $\vec{r} = \vec{a} + \lambda\vec{b}$ respectively, we get

$$\vec{a}_1 = 2\hat{i} + \hat{j} - 3\hat{k}, \vec{b}_1 = \hat{i} + 2\hat{j} + 5\hat{k}$$

and $\vec{a}_2 = 3\hat{i} + 3j + 2\hat{k}, \vec{b}_2 = 3\hat{i} - 2j + 5\hat{k}$ **(1)**

Now, the required plane which contains the lines (i) and (ii) will pass through $\vec{a}_1 = 2\hat{i} + \hat{j} - 3\hat{k}$. Also, the required plane has $\vec{b}_1$ and $\vec{b}_2$ parallel to it.

∴ The normal vector to the plane

$$\vec{n} = \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 5 \\ 3 & -2 & 5 \end{vmatrix}$$

$$= \hat{i}(10 + 10) - \hat{j}(5 - 15) + \hat{k}(-2 - 6)$$

$$= 20\hat{i} + 10\hat{j} - 8\hat{k}$$ **(1)**

∴ The vector equation of required plane is given by

$$(\vec{r} - \vec{a}) \cdot \vec{n} = 0 \Rightarrow \vec{r} \cdot \vec{n} = \vec{a}_1 \cdot \vec{n}$$

[∵ here, $\vec{a} = \vec{a}_1$]

$$\Rightarrow \vec{r} \cdot (20\hat{i} + 10\hat{j} - 8\hat{k}) = (2\hat{i} + \hat{j} - 3\hat{k}) \cdot (20\hat{i} + 10\hat{j} - 8\hat{k})$$ **(1)**

$$\Rightarrow \vec{r} \cdot (20\hat{i} + 10\hat{j} - 8\hat{k}) = 40 + 10 + 24 = 74$$

$$\Rightarrow \vec{r} \cdot (10\hat{i} + 5\hat{j} - 4\hat{k}) = 37 \text{ [dividing by 2]...(iii)}$$

which is the required equation of plane.

Also, its cartesian equation is given by

$$10x + 5y - 4z = 37$$

$\left[\because\text{ vector form of plane } \vec{r}\cdot(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) = d \text{ can be written in its cartesian form as } a_1x + a_2y + a_3z = d\right]$

Now, we have to show that the line

$$\vec{r} = (2\hat{i} + 5\hat{j} + 2\hat{k}) + P(3\hat{i} - 2\hat{j} + 5\hat{k}) \quad \text{...(iv)}$$

lies in the plane (iii). **(1)**

The above line will lies on plane (iii) when it passes through the point $\vec{a} = 2\hat{i} + 5\hat{j} + 2\hat{k}$ of line (iv) and it is parallel to line (iv).

$$\therefore \vec{a}\cdot(10\hat{i} + 5\hat{j} - 4\hat{k})$$
$$= (2\hat{i} + 5\hat{j} + 2\hat{k})\cdot(10\hat{i} + 5\hat{j} - 4\hat{k})$$
$$= 20 + 25 - 8 = 37$$

$\Rightarrow \vec{a}$ lies on plane whose equation is given by Eq. (iii).

Hence, line (iv) lies on the plane (iii). **(1)**

38. Find the equation of plane passing through the point (1, 2, 1) and perpendicular to line joining points (1, 4, 2) and (2, 3, 5).

Also, find the coordinates of foot of the perpendicular and the perpendicular distance of the point (4, 0, 3) from above found plane. **HOTS; Delhi 2011C**

Required equation of plane passing through point $R(1, 2, 1)$ and is perpendicular to line PQ, where $P(1, 4, 2)$ and $Q(2, 3, 5)$.

DR's of the line PQ are

$$= (2-1, 3-4, 5-2) = (1, -1, 3) \qquad \textbf{(1)}$$

Let the required equation of plane which passes through point $R(1, 2, 1)$ be

$$a(x-1) + b(y-2) + c(z-1) = 0 \qquad \ldots(i)$$

Plane (i) is perpendicular to line PQ.

$$[\because a_1a_2 + b_1b_2 + c_1c_2 = 0]$$

$$\therefore \quad 1(x-1) - 1(y-2) + 3(z-1) = 0$$

$$\left[\begin{array}{l}\because \text{line is perpendicular to the plane, then} \\ \text{Dr's of normal to the plane are proportional} \\ \text{to the Dr's of a line i.e. } a \propto 1, b \propto -1, c \propto 3\end{array}\right]$$

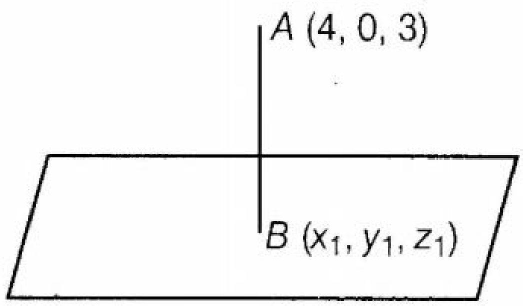
$$\Rightarrow \quad x - 1 - y + 2 + 3z - 3 = 0$$

$$\Rightarrow \quad x - y + 3z - 2 = 0 \qquad \ldots(ii)\textbf{(1)}$$

which is the required equation of plane.

Now, let $B(x_1, y_1, z_1)$ be the foot of perpendicular on above plane (ii). So, it must satisfies Eq. (ii).

$$\therefore \quad x_1 - y_1 + 3z_1 - 2 = 0 \qquad \ldots(iii)$$



(1)

Also, DR's of line AB normal to above plane (i) are given by

$$x_1 - 4, y_1 - 0, z_1 - 3$$

$$\therefore \quad \frac{x_1 - 4}{1} = \frac{y_1 - 0}{-1} = \frac{z_1 - 3}{3}$$

[∵ DR's of line AB and plane (iii) are proportional]

Let $\quad \dfrac{x_1 - 4}{1} = \dfrac{y_1}{-1} = \dfrac{z_1 - 3}{3} = \lambda \quad$ [say]

$\Rightarrow \quad x_1 = \lambda + 4, y_1 = -\lambda, z_1 = 3\lambda + 3 \quad$ ...(iv)

On putting above values of x_1, y_1 and z_1 in Eq. (iii), we get

$$\lambda + 4 + \lambda + 9\lambda + 9 - 2 = 0$$

$\Rightarrow \quad 11\lambda + 11 = 0 \Rightarrow 11\lambda = -11$

$\therefore \quad \lambda = -1 \quad$ **(1½)**

On putting $\lambda = -1$ in Eq. (iv), we get the required foot of perpendicular as

$$B(x_1, y_1, z_1) = B(\lambda + 4, -\lambda, 3\lambda + 3) = B(3, 1, 0)$$

Also, perpendicular distance AB, where $A(4, 0, 3)$ and $B(3, 1, 0)$

$$AB = \sqrt{(3-4)^2 + (1-0)^2 + (0-3)^2}$$

$$\left[\because \text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}\right]$$

$= \sqrt{1 + 1 + 9} = \sqrt{11}$ units $\quad$ **(1½)**

NOTE If line is perpendicular to the plane, then DR's of normal to the plane are proportional to the DR's of a line.

39. Find the equation of plane passing through point $P(1, 1, 1)$ and containing the line $\vec{r} = (-3\hat{i} + \hat{j} + 5\hat{k}) + \lambda(3\hat{i} - \hat{j} - 5\hat{k})$. Also, show that plane contains the line $\vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \mu(\hat{i} - 2\hat{j} - 5\hat{k})$.

All India 2010

Equation of plane passing through point $P(1, 1, 1)$ is given by

$a(x-1) + b(y-1) + c(z-1) = 0$ ...(i)**(1)**

[$\because$ equation of plane passing through (x_1, y_1, z_1) is given as $a(x-x_1) + b(y-y_1) + c(z-z_1) = 0$]

Given equation of line is

$$\vec{r} = (-3\hat{i} + \hat{j} + 5\hat{k}) + \lambda(3\hat{i} - \hat{j} - 5\hat{k}) \quad \text{...(ii)}$$

DR's of the line are 3, –1 and –5 and the line passes through point (–3, 1, 5).

Now, as the plane (i) contains line (ii), so

$$a(-3-1) + b(1-1) + c(5-1) = 0$$

$\left[\begin{array}{l}\text{as plane contain a line, it means point} \\ \text{of line lie on a plane.}\end{array}\right]$

$$\Rightarrow \quad -4a + 4c = 0 \quad \text{...(iii)} \quad \textbf{(1)}$$

$$\Rightarrow \quad 4a = 4c$$

$$\Rightarrow \quad a = c$$

Also, since DR's of plane are normal to that of line.

$$\therefore \quad 3a - b - 5c = 0 \quad \text{...(iv)}$$

[$\because$ plane contains line, it means DR's of plane are perpendicular to the line, i.e. $aa_1 + bb_1 + cc_1 = 0$]

On putting $a = c$ in Eq. (iv), we get

$$3c - b - 5c = 0 \quad \textbf{(1)}$$

$$\Rightarrow \quad -b - 2c = 0$$

$$\Rightarrow \quad b = -2c$$

On putting $a = c$ and $b = -2c$ in Eq.(i), we get the required equation of plane as

$$c(x-1) - 2c(y-1) + c(z-1) = 0$$

On dividing both sides by c, we get

$$x - 1 - 2y + 2 + z - 1 = 0$$

$$\Rightarrow \quad x - 2y + z = 0 \quad \text{...(v)}\textbf{(1½)}$$

Now, we have to show that the above plane

Now, we have to show that the above plane (v) contains the line

$$\vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \mu(\hat{i} - 2\hat{j} - 5\hat{k}) \quad \text{...(vi)}$$

Vector equation of plane (v) is

$$\vec{r} \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 0 \quad \text{...(vii)}$$

The plane (vii) will contains line (vi), if

$$(\hat{i} - 2\hat{j} - 5\hat{k}) \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 0 \quad [\because \vec{b}_1 \cdot \vec{b}_2 = 0]$$

$\Rightarrow$ (1) (1) − 2 (−2) − 5 (1) = 0

$\Rightarrow$ 1 + 4 − 5 = 0

$\Rightarrow$ 0 = 0, which is true. **(1½)**

Hence, the plane contains the given line.

40. Find the coordinates of the foot of perpendicular and the perpendicular distance of point $P(3, 2, 1)$ from the plane $2x - y + z + 1 = 0$. Also, find image of the point in the plane. **All India 2010**

Do same as Q. 31.

$$\left[\begin{array}{l}\textbf{Ans.} \text{ Foot of perpendicular } = (1, 3, 0), \\ \text{perpendicular distance } = \sqrt{6} \text{ units} \\ \text{Image point } = (-1, 4, -1)\end{array}\right]$$

41. Find the distance of the point (2, 3, 4) from the line $\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$ measured parallel to the plane $3x + 2y + 2z - 5 = 0$.

HOTS ; All India 2009C

Let $P(2, 3, 4)$ be the given point and given equation of line be

$$\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$$

Let $$\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2} = \lambda \quad \text{[say]}$$

$\Rightarrow \quad x = 3\lambda - 3, y = 6\lambda + 2, z = 2\lambda$

$\therefore$ Coordinates of any point T on given line are $(3\lambda - 3, 6\lambda + 2, 2\lambda)$. **(1½)**

Now, DR's of line PT

$$= (3\lambda - 3 - 2, 6\lambda + 2 - 3, 2\lambda - 4)$$
$$= (3\lambda - 5, 6\lambda - 1, 2\lambda - 4) \quad \textbf{(1)}$$

Since, the line PT is parallel to the plane

$$3x + 2y + 2z - 5 = 0$$

$\therefore \quad a_1a_2 + b_1b_2 + c_1c_2 = 0$

[$\because$ line is parallel to the plane, therefore normal to the plane is perpendicular to the line, where $a_1 = 3\lambda - 5, b_1 = 6\lambda - 1, c_1 = 2\lambda - 4$ and $a_2 = 3, b_2 = 2, c_2 = 2$]

$\Rightarrow 3(3\lambda - 5) + 2(6\lambda - 1) + 2(2\lambda - 4) = 0$ **(1½)**

$\Rightarrow \quad 9\lambda - 15 + 12\lambda - 2 + 4\lambda - 8 = 0$

$\Rightarrow \quad 25\lambda - 25 = 0 \Rightarrow 25\lambda = 25$

$\Rightarrow \quad \lambda = 1$ **(1)**

$\therefore$ Coordinates of $T = (3\lambda - 3, 6\lambda + 2, 2\lambda)$

$= (0, 8, 2) \quad$ [$\because$ put $\lambda = 1$]

Now, the required distance between points $P(2, 3, 4)$ and $T(0, 8, 2)$ is given by

$$PT = \sqrt{(0-2)^2 + (8-3)^2 + (2-4)^2}$$

[$\because (x_1, y_1, z_1) = (2, 3, 4)$ and $(x_2, y_2, z_2) = (0, 8, 2)$]

$$= \sqrt{4 + 25 + 4} = \sqrt{33} \text{ units} \quad \textbf{(1)}$$

42. Find the distance of point (–2, 3, –4) from the line $\frac{x+2}{3}=\frac{2y+3}{4}=\frac{3z+4}{5}$ measured parallel to the plane $4x+12y-3z+1=0$.

HOTS; All India 2009C, 2008C, 2008

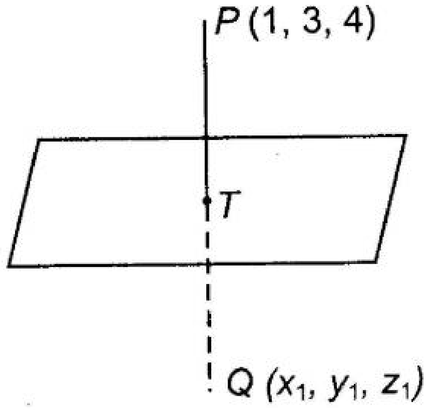
Do same as Que. 41. $\left[\textbf{Ans. } \frac{17}{2} \text{ units}\right]$

43. Find the coordinates of image of point (1, 3, 4) in the plane $2x-y+z+3=0$.

All India 2008

Let $Q(x_1, y_1, z_1)$ be the image of the point $P(1, 3, 4)$ on the plane whose equation is

$$2x-y+z+3=0 \quad \ldots\text{(i)}$$



(1)

Now, the line PT is normal to the plane, so the DR's of PT are proportional to the DR's of plane which are 2, –1 and 1.

$\therefore$ Equation of line PT, where $P(1, 3, 4)$ is given by

$$\frac{x-1}{2}=\frac{y-3}{-1}=\frac{z-4}{1} \quad \ldots\text{(ii)}\textbf{(1)}$$

$\left[\because \text{ equation of line passing through one point is } \frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}\right]$

Let $\frac{x-1}{2}=\frac{y-3}{-1}=\frac{z-4}{1}=\lambda$

$\Rightarrow$ $x=2\lambda+1, y=3-\lambda, z=4+\lambda$

∴ Coordinates of the point T

$$= (2\lambda + 1, 3 - \lambda, 4 + \lambda) \quad ...(iii)$$

Since, the point T lies on the plane, so we put $x = 2\lambda + 1, y = 3 - \lambda$ and $z = 4 + \lambda$ in Eq. (i),

we get $2(2\lambda + 1) - (3 - \lambda) + (4 + \lambda) + 3 = 0$

$\Rightarrow \quad 4\lambda + 2 - 3 + \lambda + 4 + \lambda + 3 = 0$

$\Rightarrow \quad 6\lambda + 6 = 0$

$\Rightarrow \quad 6\lambda = -6$

$\therefore \quad \lambda = -1$ **(1½)**

On putting $\lambda = -1$ in Eq. (iii), we get the point $T(-1, 4, 3)$.

Now, T is the mid-point of line PQ. So, using mid-point formula, we get

$$\left(\frac{x_1 + 1}{2}, \frac{y_1 + 3}{2}, \frac{z_1 + 4}{2}\right) = (-1, 4, 3) \quad \textbf{(1)}$$

$$\left[\because \text{mid-point} = \left(\frac{x_1 + x_2}{2} = \frac{y_1 + y_2}{2} = \frac{z_1 + z_2}{2}\right)\right]$$

On equating corresponding coordinates, we get

$$\frac{x_1 + 1}{2} = -1, \frac{y_1 + 3}{2} = 4, \frac{z_1 + 4}{2} = 3$$

$\Rightarrow \quad x_1 = -2 - 1, y_1 = 8 - 3, z_1 = 6 - 4$

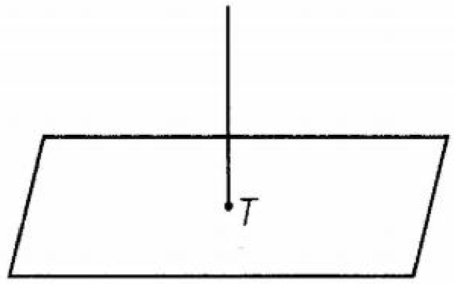
$\Rightarrow \quad x_1 = -3, y_1 = 5, z_1 = 2$

Hence, image Q of the point $P(1, 3, 4)$ is $Q(-3, 5, 2)$. **(1½)**

44. From the point $P(1, 2, 4)$, a perpendicular is drawn on the plane $2x + y - 2z + 3 = 0$. Find the equation, the length and the coordinates of foot of perpendicular. **All India 2008**

Let PT be the perpendicular drawn from the point $P(1, 2, 4)$ to the plane whose equation is given by

$$2x + y - 2z + 3 = 0 \quad \text{...(i)}$$



From Eq. (i), DR's of plane are 2, 1, −2.
Since, the line PT is normal to the plane, so DR's of line normal to plane are 2, 1, −2. **(1)**

∴ Equation of line PT,
where $P(1, 2, 4)$ is given as

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-4}{-2} \quad \text{...(ii)}$$

[∵ equation of line passing through one point is $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$]

Let $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-4}{-2} = \lambda$ [say]

$\Rightarrow \frac{x-1}{2} = \lambda, \frac{y-2}{1} = \lambda, \frac{z-4}{-2} = \lambda$

$\Rightarrow x = 2\lambda + 1, y = \lambda + 2, z = -2\lambda + 4$

∴ Coordinates of any random point on plane are

$T(2\lambda + 1, \lambda + 2, -2\lambda + 4)$ ...(iii) **(1½)**

Since, T lies on the given plane, so we put $x = 2\lambda + 1, y = \lambda + 2$ and $z = -2\lambda + 4$ in Eq. (i), we get

$2(2\lambda + 1) + (\lambda + 2) - 2(-2\lambda + 4) + 3 = 0$

$\Rightarrow 4\lambda + 2 + \lambda + 2 + 4\lambda - 8 + 3 = 0$

$\Rightarrow 9\lambda - 1 = 0$

$\therefore \lambda = \frac{1}{-}$ **(1½)**

On putting value of λ in Eq. (iii), we get the foot of perpendicular

$$= T\left(\frac{2}{9}+1, \frac{1}{9}+2, -\frac{2}{9}+4\right) \left[\therefore \text{ put } \lambda = \frac{1}{9}\right]$$

$$= T\left(\frac{11}{9}, \frac{19}{9}, \frac{34}{9}\right)$$

Also, length of perpendicular PT = Distance between points P and T

$$= \sqrt{\left(1-\frac{11}{9}\right)^2 + \left(2-\frac{19}{9}\right)^2 + \left(4-\frac{34}{9}\right)^2}$$

$$\left[\begin{array}{l} \because (x_1, y_1, z_1) = (1, 2, 4) \\ \text{and } (x_2, y_2, z_2) = \left(\frac{11}{9}, \frac{19}{9}, \frac{34}{9}\right) \end{array}\right]$$

$$= \sqrt{\left(-\frac{2}{9}\right)^2 + \left(-\frac{1}{9}\right)^2 + \left(\frac{2}{9}\right)^2}$$

$$= \sqrt{\frac{4}{81}+\frac{1}{81}+\frac{4}{81}} = \sqrt{\frac{9}{81}} = \sqrt{\frac{1}{9}} = \frac{1}{3} \text{ unit} \quad \textbf{(1)}$$

Now, the equation of perpendicular line PT, where $P(1, 2, 4)$ and $T\left(\frac{11}{9}, \frac{19}{9}, \frac{34}{9}\right)$ is given as

$$\frac{x-1}{\frac{11}{9}-1} = \frac{y-2}{\frac{19}{9}-2} = \frac{z-4}{\frac{34}{9}-4}$$

$$\left[\begin{array}{l} \because \text{ using two points form of a line} \\ \frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1} \end{array}\right]$$

$$\Rightarrow \quad \frac{x-1}{\left(\frac{2}{9}\right)} = \frac{y-2}{\left(\frac{1}{9}\right)} = \frac{z-4}{\left(-\frac{2}{9}\right)}$$

$$\therefore \quad \frac{x-1}{2} = \frac{y-2}{1} = \frac{z-4}{-2} \quad \textbf{(1)}$$

45. Find the equation of plane passing through the point (–1, –1, 2) and perpendicular to each plane $2x + 3y - 3z = 2$ and

$$5x - 4y + z = 6.$$ **Delhi 2008**

Do same as Que. 34.

[Ans. $9x + 17y + 23z - 20 = 0$]

46. Find the equation of plane passing through points (3, 4, 1) and (0, 1, 0) and parallel to line $\frac{x+3}{2} = \frac{y-3}{7} = \frac{z-2}{5}$. **HOTS; Delhi 2008**

Equation of plane passing through the point (3, 4, 1) is given as

$$a(x-3) + b(y-4) + c(z-1) = 0 \quad \text{...(i)}$$

[∵ using one point form of a plane

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0]$$

Since, given plane (i) is also passing through point (0, 1, 0).

So, this point also satisfies equation of plane.

$\therefore \quad a(0-3) + b(1-4) + c(0-1) = 0$

$\Rightarrow \quad -3a - 3b - c = 0$

$\Rightarrow \quad 3a + 3b + c = 0$ ...(ii) **(1)**

Also, given that plane (i) is parallel to the line

$$\frac{x+3}{2} = \frac{y-3}{7} = \frac{z-2}{5}$$

[∴ line is parallel to the plane, therefore normal to the plane is perpendicular to the line, i.e. $a_1a_2 + b_1b_2 + c_1c_2 = 0$
where, $a_1 = a, b_1 = b, c_1 = c$
and $a_2 = 2, b_2 = 7$ and $c_2 = 5$]

$\therefore \quad 2a + 7b + 5c = 0$ ...(iii) **(1½)**

On multiplying Eq. (ii) by 2 and Eq. (iii) by 3 and then subtracting, we get

$$6a + 6b + 2c = 0$$

$$6a + 21b + 15c = 0$$

$$-15b - 13c = 0$$

$\Rightarrow$ $$-15b = 13c$$

$\therefore$ $$b = \frac{-13}{15}c$$

On putting $b = \frac{-13c}{15}$ in Eq. (ii), we get

$$3a + 3\left(\frac{-13c}{15}\right) + c = 0$$

$\Rightarrow$ $$3a - \frac{13}{5}c + c = 0$$

$\Rightarrow$ $$3a - \frac{8c}{5} = 0$$

$\Rightarrow$ $$3a = \frac{8c}{5}$$

$\therefore$ $$a = \frac{8c}{15}$$ **(1½)**

On putting $a = \frac{8c}{15}$ and $b = -\frac{13c}{15}$ in Eq. (i),

we get the required equation of plane as

$$\frac{8c}{15}(x-3) - \frac{13c}{15}(y-4) + c(z-1) = 0$$ **(1)**

On dividing both sides by c, we get

$$\frac{8}{15}(x-3) - \frac{13}{15}(y-4) + z - 1 = 0$$

$\Rightarrow$ $$8x - 24 - 13y + 52 + 15z - 15 = 0$$

$\therefore$ $$8x - 13y + 15z + 13 = 0$$ **(1)**